10th International Young Researcher Workshop on Geometry, Mechanics and Control

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Posters

1. Local synthesis of 2D and 3D sub-Finslerian metrics

Entisar Abdul-Latif ALI
Institut Fourier, France.

I will give local properties of optimal extremals for generic 2D and 3D-contact sub-Finslerian metrics, give a complete description of the synthesis in the generic 2D case, and first results on the cut locus and "conjugate" locus in the generic 3D-contact case.

2. Controllability of Keplerian Motion with Low-Thrust Control Systems

Zheng CHEN
Université Paris-Sud, France.

We present the controllability properties of the Keplerian motion controlled by low-thrust control systems. The low-thrust control system, compared with high or even impulsive control system, provide a fuel-efficient means to control the Keplerian motion of a satellite in restricted two-body problem. We obtain that, for any positive value of maximum thrust, the motion is controllable for orbital transfer problems. For two other typical problems: de-orbit problem and orbital insertion problem, which have state constraints, the motion is controllable if and only if the maximum thrust is bigger than a limiting value. Finally, a numerical method to compute the limiting value is presented.

3. On strong automorphisms of regular Courant algebroids

Benjamin COUERAUD
Université d'Angers, France.

Since the work of Courant on the integrability of Dirac structures, and that of Liu, Weinstein and Xu on Lie bialgebroids, Courant algebroids have become important objects in Differential Geometry and Theoretical Physics. Exact Courant algebroids are of particular interest; for example, they play a fundamental role in generalized complex geometry where, in addition to diffeomorphisms of the underlying manifold, new symmetries appear, given by the so-called B-fields, which are 2-forms. Such a description is obtained using a splitting of the short exact sequence defining an exact Courant algebroid. Chen, Stiénon and Xu have introduced the appropriate generalization of such a splitting for regular Courant algebroids, called a dissection. Using a dissection of a regular Courant algebroid, we describe explicitly both the global and the infinitesimal automorphisms. New symmetries appear, the so-called A-fields, which are 1-forms taking values in a quadratic Lie algebra bundle associated to the algebroid; these fields are of interest in both Mathematics and Theoretical Physics. This result encompasses previous results from D_n-geometry, B_n-geometry and heterotic Courant algebroids.

4. Multisymplectic unified formalism for the Hilbert-Einstein Lagrangian

Jordi GASET
Universitat Politècnica de Catalunya.

In recent years a multisymplectic approach to General Relativity using first order Lagrangians has ben developed [1], [2], [3]. Using a recent work in second order field theories [4], we apply the unified formalism to the original second-order Hilbert-Einstein Lagrangian. The unified formalism will give us a new view on the degeneracy, the symmetries and the Hamiltonian formalism of General Relativity.

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[2] E. Rosado María, J. Muñoz Masqué, Second-Order Lagrangians admitting a first-order Hamiltonian formalism, arXiv:1509.01037 (2015)
[3] D. Vey, Multisymplectic formulation of vielbein gravity. De Donder-Weyl formulation, Hamiltonian $(n-1)$-forms, Quantum Grav. 32 095005, 2015.
[4] P. D. Prieto-Martínez, N. Román-Roy, A new multisymplectic unified formalism for second order classical field theories, Journal of Geometric Mechanics 7 (2), 203 - 253.

5. TBA

Mohammed Amine GHEZZAR
UMAB-Mostaganem/UPJV-Amiens

6. Cyclic translating solitons in $\mathbb{R}^3$

Daehwan KIM
Pusan National University in the Republic of Korea

We deal with the translating soliton foliated by circles in $\mathbb{R}^3$. A surface is called a translating soliton if it translates with constant velocity v without changing its shape under the mean curvature flow. It is satisfied the equation: $H = \langle n, v \rangle$ where H and n are the mean curvature and a unit normal vector field of the surface, respectively. Up to isometry, we can assume v equals to e_3. When a surface foliated by circles is called the cyclic surface, we prove that a cyclic translating soliton is a surface of revolution with the axis of revolution is parallel to e_3.

7. TBA

Saliha MARIR
UMAB-Mostaganem/UPJV-Amiens

8. Non-diagonal metric on a product Riemannian manifold.

Rafik NASRI
Tahar Moulay University, Algeria

In this work, we construct the symmetric tensor field $G_{f_1 f_2}$ and $h_{f_1 f_2}$ on a product manifold and we give conditions under which $G_{f_1 f_2}$ becomes a metric tensor, theses tensors fields will be called the generalized warped product, and then we develop an expression of curvature for the connection of the generalized warped product in relation to those corresponding analogues of its base and fiber and warping functions. By constructing a frame field in $M_1 \times_{f_1 f_2} M_2$ with respect to the Riemannian metric $G_{f_1 f_2}$ and $h_{f_1 f_2}$, then we calculate the Laplacian−Beltrami operator of a function on a generalized

warped product which may be expressed in terms of the local restrictions of the functions to the base and fiber. Finally, we conclude some interesting relationships between the geometry of the couples (M_1, g_1) and (M_2, g_2) and that of $(M_1 \times M_2, h_{f_1 f_2})$

9. A blow-up criteria and local well-posedness for gravity-capillary water waves at low regularity

Quang Huy NGUYEN
Université Paris-Sud, France

In fluid dynamics, the gravity-capillary water waves system is used to describe the motion of an interface between a fluid and the vacuum, under gravity force and surface tension. An important question in the analysis of this Hamiltonian system is: how do solutions break down? First, we prove a blow-up criteria for solutions based on following geometric and physical quantities: the boundedness of curvature of the interface and the Lipschitzity of the velocity field. Then applying the preceding result together with a dispersive estimate we construct solutions for initial velocities that may fail to be Lipschitz. This is a joint work with Thibault de Poyerré (ENS, Paris) in arXiv:1508.00326 and arXiv:1507.08918, 2015.

10. Biharmonic maps between some particular manifolds

Seddik OUAKKAS
Université de Saida, Algeria

Let $\phi : (M^m, g) \to (N^n, h)$ be a smooth map between Riemannian manifolds. Then ϕ is said to be harmonic if it is a critical point of the energy functional :

$$E(\phi) = \frac{1}{2} \int_M |d\phi|^2 dv_g$$

with respect to compactly supported variations. Equivalently, ϕ is harmonic if it satisfies the associated Euler-Lagrange equations

$$\tau(\phi) = Tr_g \nabla d\phi = 0,$$

$\tau(\phi)$ is called the tension field of ϕ. Indeed, the Euler-Lagrange equation associated to the energy is the vanishing of the tension field $\tau(\phi) = Tr_g \nabla d\phi$.

The map ϕ is said to be biharmonic if it is a critical point of the bi-energy functional

$$E_2(\phi) = \frac{1}{2} \int_M |\tau(\phi)|^2 dv_g.$$

Equivalently, ϕ is biharmonic if it satisfies the associated Euler-Lagrange equations :

$$\tau_2(\phi) = -Tr_g(\nabla^\phi)^2\tau(\phi) - Tr_g R^N(\tau(\phi), d\phi)d\phi = 0,$$

where ∇^ϕ is the connection in the pull-back bundle $\phi^{-1}(TN)$ and, if e_i is a local orthonormal frame field on M, then

$$Tr_g(\nabla^\phi)^2\tau(\phi) = \left(\nabla^\phi_{e_i}\nabla^\phi_{e_i} - \nabla^\phi_{\nabla_{e_i}e_i}\right)\tau(\phi),$$

where we sum over repeated indices. We will call the operator $\tau_2(\phi)$ the bi-tension field of the map ϕ.

In this work, we give some constructions for harmonic and biharmonic maps between some particular manifolds when we give the necessary and sufficient conditions. This study allowed us to construct new examples.

11. On the definition and regularity of higher-order Hamiltonian functions

Pedro Daniel PRIETO-MARTINEZ
Universitat Politècnica de Catalunya

The aim of this talk is to generalize some results from [1], namely Propositions 3.6.7 and 3.6.8, and Theorem 3.6.9, to higher-order (autonomous) dynamical systems. The attempt to generalize these results gives rise to another more interesting problem: what is a "higher-order Hamiltonian function"? And what does it mean for such a function to be "regular"? These concepts are clearly defined for first-order Lagrangian or Hamiltonian functions, and also for a higher-order Lagrangian function. In this talk we propose a definition for such concepts, always taking into account the particular case of Hamiltonian functions associated to a (regular) Lagrangian system.

References

[1] R. Abraham and J.E. Marsden, Foundations of mechanics, 2nd ed., Benjamin-Cummings, New York, 1978.
[2] M. de León and P.R. Rodrigues, Generalized classical mechanics and field theory, North-Holland Math. Studies, vol. 112, Elsevier Science Publishers B.V., Amsterdam 1985.

12. Constraint algorithm for singular k-cosymplectic field theories

Xavier RIVAS
Universitat Politècnica de Catalunya

Nonautonomous classical field theories can be described using k-cosymplectic geometry. In particular, those theories described by singular lagrangian are of special interest because of their role in modern physics. The systems of PDEs appearing in these systems (Euler-Lagrange and Hamilton-de Donder-Weyl) have a problem: the incompatibility of solutions. However, sometimes there exist submanifolds of the manifold where we have the equations defined, where we can find consistent solutions. In this poster we define the concept of k-precosymplectic manifold, prove the existence of Reeb vector fields and develop a geometric constraint algorithm in order to find a constraint submanifold where we have assured the existence of solutions of singular k-cosymplectic field theories.

13. From graded bundles to k-tuple vector bundles and back

Mikolaj ROTKIEWICZ
Warsaw University

On the notion of the *full linearisation* as functor from the category of graded bundles of degree k to the category of k-tuple vector bundles. In the standard example the functor associates $TT\dots TM$ to T^kM. Application to higher order mechanics are possible.

14. Lie–Hamilton systems on curved spaces

Mariusz TOBOLSKI
University of Warsaw

Lie–Hamilton systems are non-autonomous systems of first-order ordinary differential equations whose dynamic is determined by a curve in a finite-dimensional Lie algebra of Hamiltonian vector fields. The general solution of a Lie–Hamilton system can be described as a function, called a superposition rule, of a finite family of particular solutions and some constants. We briefly survey the classification of finite-dimensional Lie algebras of Hamiltonian vector fields on the plane and its use to study planar Lie–Hamilton systems and their superposition rules [1, 2]. Next, we generalise and analyse the Lie–Hamilton system on the plane of type P_1 to the 2-dimensional curved spacetimes, the so-called Cayley-Klein spaces. We also give superposition rules for these new

Lie–Hamilton systems using unified trigonometry formulas [3], which provide a clear geometric understanding of superposition rules.

[1] A. Ballesteros, A. Blasco, F.J. Herranz and C. Sard on, Lie–Hamilton systems on the plane: properties, classification and applications, J. Differential Equations 258, 2873-2907 (2015).
[2] A. Ballesteros, A. Blasco, F.J. Herranz and J. de Lucas, Lie–Hamilton systems on the plane: applications and superposition rules, J. Phys. A48, 345202 (2013). [3] F.J. Herranz, R. Ortega and M. Santander, Trigonometry of spacetimes: a new self-dual approach to a curvature/signature (in)dependent trigonometry, J. Phys. A33, 4525-4551 (2000).

15. The Tulczyjew triple in mechanics on a Lie group

Marcin ZAJAC
University of Warsaw, Poland.

Recently the Tulczyjew triple became a very important tool in geometrical description of classical mechanics. There are many versions of the Tulczyjew triple described in literature, in particular classical one and its algebroidal generalization. Both of them are well known and widely discussed in many papers.

In the talk I will discuss the Tulczyjew formalism in case when a configuration manifold of the system is a Lie group. The tangent bundle to a Lie group has a natural structure of a trivial bundle, therefore whole picture of generating the dynamics may be trivialized. In particular, it requires trivializing canonical isomorphisms of double vector bundles present in a Tulczyjew triple. I will also consider reduction of the Tulczyjew triple with respect to the group action.

In the end I will show a physical application of derived formalism, in particular how reduced Tulczyjew triple may simplify description of the system with lagrangian invariant under the group action.

Description of the group-case triple fills the gap beetween its classical and algebroidal version.